Lateral buckling of steel plate girders for bridges with flexible lateral restraints or torsional restraints

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Abstract

Since publication of BS 5400:Part 3:2000, the design check of paired plate girders during erection has become more onerous in the UK. This has led to increases in top flange size or the provision of plan bracing just for the erection condition where previously neither modification to the permanent design was likely to be needed. This change has been brought about by the addition of two main features in BS 5400:Part 3:2000; a change to the mode of buckling considered in deriving the girder slenderness and the addition of more conservative buckling curves when the effective length for buckling differs from the half wavelength of buckling. The latter change was incorporated because of concerns that the imperfection over the half wavelength was more relevant than that over the effective length which is implicit in traditional strut curves. BS EN 1993 Part 1-1 requires no such reduction in resistance and the work in this paper was prompted by a proposal to modify the rules of BS EN 1993-2 for bridges in the UK's National Annex to be more like those in BS 5400:Part 3. The authors believed this to be unnecessary.

This paper investigates buckling cases where the effective length for buckling is shorter than the half wavelength of buckling and demonstrates that the series of correction curves use in BS 5400:Part 3:2000 are unnecessary and that the BS EN 1993-1-1 method is satisfactory and slightly conservative. The paper also outlines the design process to BS EN 1993 using both elastic critical buckling analysis and non-linear analysis. The case studies considered are a simple pin-ended strut with intermediate restraints, a pair of braced girders prior to hardening of the deck slab and a half-through deck with discrete U-frame restraints. For the latter two cases, the results predicted by BS 5400:Part 3 and BS EN 1993-1-1 are compared with the results of non-linear finite element analyses.

1. Introduction

An important part of the design of a steel concrete composite bridge is the stability check of the girders during construction of the concrete slab and prior to it hardening, whereupon it provides continuous restraint to the top flange. This may be a critical check as the girders will often be most susceptible to lateral torsional buckling (LTB) failure when the deck slab is being poured. Beams are normally braced in pairs with discrete torsional restraints, often in the form of X bracing or K bracing (as shown in Figure 1), but for shallower girders single horizontal channels connecting the beams at mid-height is an economic, but less rigid, alternative.

Paired girders with torsional bracing generally fail by rotation of the braced pair over a span length as shown in Figure 2. With widely spaced torsional bracing, buckling of the compression flange between bracing points is also possible. It was once thought that torsional bracing was effective in limiting failure to occur by buckling of the compression flange between restraints but this is now known not to be so and is reflected in the calculation method given in BS 5400:Part 3:2000[1].

FIGURE 1. BRACED BEAMS IN PAIRS PRIOR TO CONCRETING

The previous incorrect approach however allowed girders to be constructed safely for many years, probably due to incidental bracing arising from frictional restraint of the formwork and because of partial factors used in design. The mode shown in Figure 2 is however prevented by adding plan bracing to the compression flange (which is effectively provided by the deck slab once it hardens) and the latter mode (buckling of the flange between bracings) then occurs. If the check of the paired beams during concreting suggests inadequacy, either the compression flange has to be increased in size or plan bracing added.

Plan bracing is not a popular choice with contractors in the UK. If the bracing is placed above the top flange for incorporation within the slab, it interferes with reinforcement fixing and the permanent formwork. If it is placed to the underside of the top flange, it presents both a long term maintenance liability and a short term health and safety hazard during its erection. It is often therefore preferred to increase the width of the top flange or provide more discrete torsional bracing.

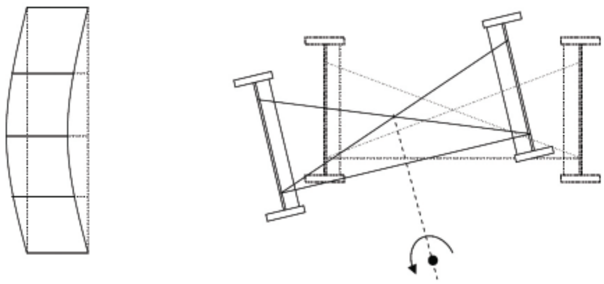


FIGURE 2. BUCKLING OF PAIRED BEAMS PRIOR TO CONCRETE HARDENING

The calculation of buckling resistance for the construction condition is currently both lengthy and conservative to BS 5400:Part 3:2000. This has the consequence that frequently the check is not carried out properly at the tender stage of a project. When the check is subsequently carried out at the detailed design stage, it is often found to require additional bracing or changes to plate thickness. One of the reasons for this is that the current BS 5400:Part 3 method is too conservative.

The design method for beams with discrete torsional restraints (the construction condition above) in BS 5400:Part 3:2000 (and BD 13/06[2]) is very conservative for a number of reasons as follows:

(i) The use of multiple strength-slenderness curves for different l_e/l_w ratios, which take the imperfection appropriate to the half wavelength of buckling, lw, (typically the span length) and apply it to the shorter effective length, l_e, is incorrect and this is demonstrated in the remainder of this paper.

(ii) The calculation of effective length for the true buckling mode is simplified and conservative. To overcome this, an elastic critical buckling analysis can be performed to determine the elastic critical buckling moment and hence slenderness. This technique is used later in this paper.

(iii) The curves provided to relate strength reduction factor to slenderness, which are derived for strut buckling, are slightly conservative for a mode of buckling where the paired girders buckle together by a combination of opposing bending vertically and lateral bending of the flanges.

(iv) Incidental frictional restraint from formwork is ignored.

The first of these issues is studied in the remainder of this paper and it is shown that the current multiple strength-slenderness curves for different l_e/l_w ratios in BS 5400:Part 3 are incorrect and overly conservative. This conclusion applies both to the construction condition above and to beams with U-frame support to the compression flange, since both cases produce an effective length that is less than the half wavelength of buckling. Non-linear analysis is used to illustrate this conclusion.

2. Buckling curves

The buckling curves in BS 5400:Part 3:2000 are based on a pin ended strut with the half wavelength of buckling equal to the effective length of the strut. The resistance is dependent on the initial geometric imperfection assumed and the residual stresses in the section. The equivalent geometric imperfection implicit in these equations is not constant but is slenderness-dependent (and therefore a function of effective length) in order to produce a good fit with test results.

The buckling curves in BS 5400:Part 3:2000 for beams with intermediate restraints are modified based on the ratio between the effective length, l_e, and the half wavelength

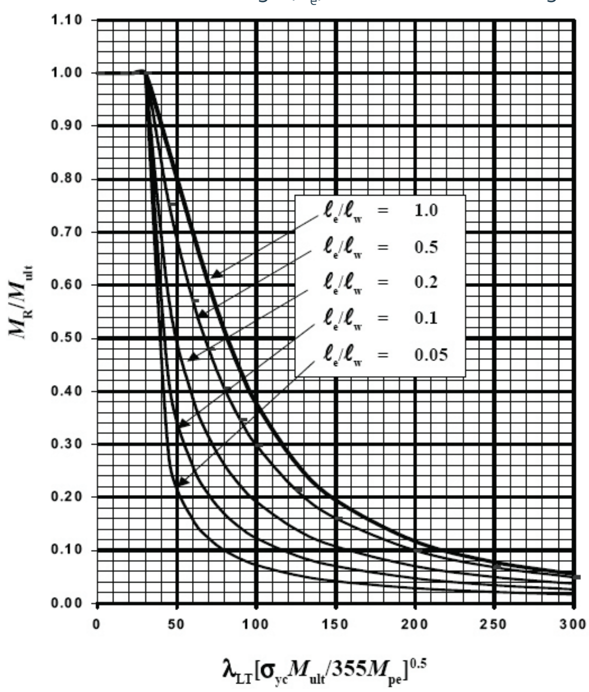


FIGURE 3. BD 13/06 BUCKLING CURVES (FOR WELDED MEMBERS)

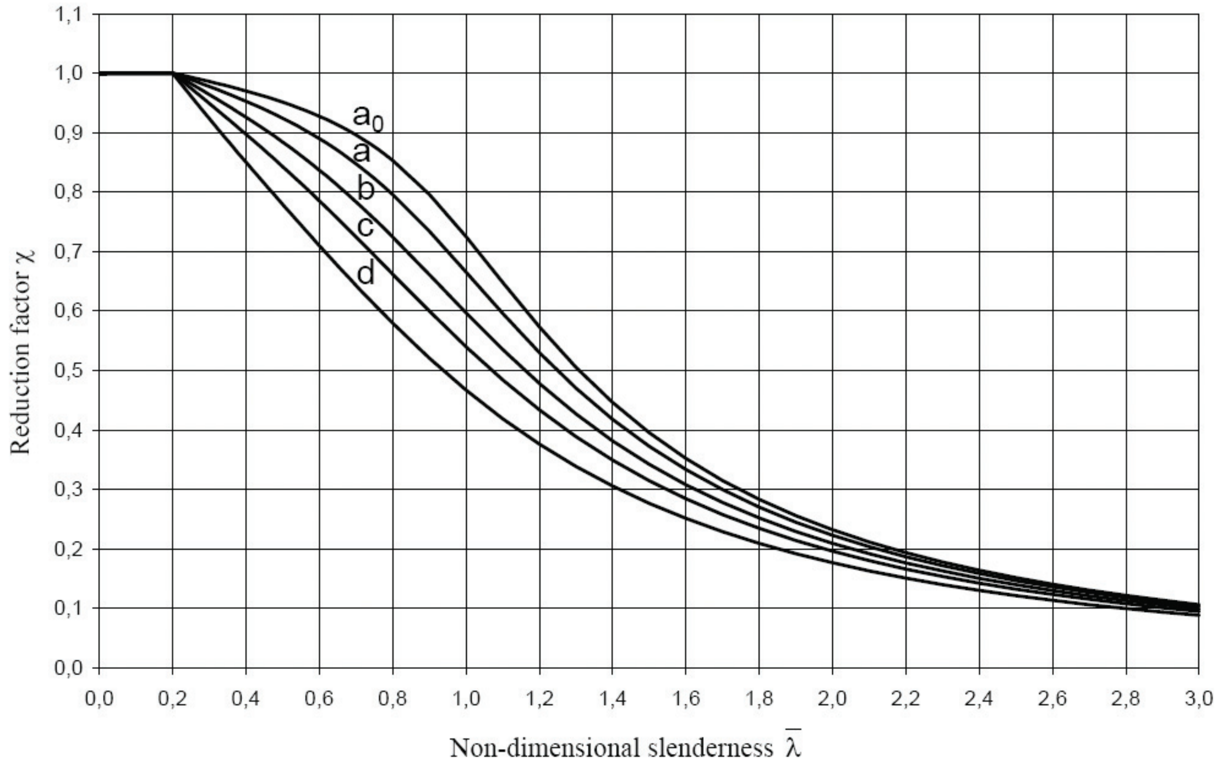


FIGURE 4. BS EN 1993-1-1 BUCKLING CURVES (A – D REFER TO THE FABRICATION METHOD)

of buckling, l_w. This was considered necessary in order to factor up the imperfection to be used in the buckling curve from that appropriate to the effective length to that appropriate to the half wavelength of buckling. BS 5400:Part 3 gives rules for calculating l_e where lateral restraints to the compression flange, torsional restraints, discrete U-frame restraints or restraint from the bridge deck are provided. Where lateral restraints to the compression flange are fully effective, l_e is taken as the span between restraints, and l_w will also be equal to this span. Where the restraints are not fully effective, l_e may be shorter than l. This is usually the case for beams relying on U-frame restraint or for paired beams with torsional bracing like that shown in Figure 1. BS 5400:Part 3 states that l_w is determined by taking L/l_w as the next integer below L/l_e where L is the span of the beam between supports. The rules in BS 5400:Part 3 are modified in BD 13/06 for use on Highways Agency projects. BD 13/06 introduces further conservatism for beams with torsional restraints by requiring lw to be taken as the full span. The buckling curves for welded members in BD13/06 are shown in Figure 3.

The buckling curves in BS EN 1993-1-1[3] (see Figure 4) have the same basis as, and are effectively the same as, those in BS 5400, but no adjustment is made for the ratio l_e/l. A non-dimensional presentation of slenderness is also used.

3. Buckling cases investigated

To investigate the validity of the BS 5400:Part 3 approach for beams with lateral restraints, a number of situations were considered. First, to clarify the principles involved, a pin-ended strut with springs providing lateral restraints was considered. The behaviour of this simple model, axially loaded and with an initial geometric imperfection, was compared to the behaviour of an equivalent strut with no lateral restraints but the same elastic critical buckling load. Second, two practical situations where a reduction in capacity to BS 5400:Part 3 would be required due to differences between the half wavelength of buckling and the effective length were considered. These were a typical half-through bridge with U-frames and a steel-concrete composite bridge during construction.

3.1 Simple strut model

Figure 5 illustrates two struts with identical elastic critical buckling loads; one with flexible intermediate transverse restraints and the other without. BS 5400:Part 3 and BD 13/06 would predict the case with intermediate restraints to have the lower ultimate resistance (as distinct from elastic critical buckling load) because it has a ratio of $l_e/l_w < 1.0$. The comparison of true ultimate strength in the two cases was examined.

A 10 m long strut in S355 steel with springs at 1m centres as shown in Figure 6 was considered. The strut was pinned at either end.

The cross section was square with 100 mm sides and the springs had a stiffness of 10 kNm^{-1}.

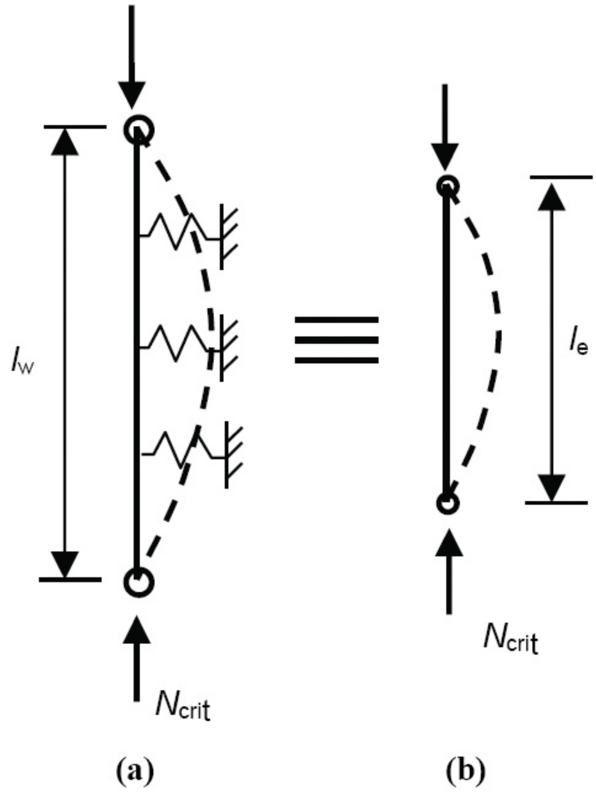


FIGURE 5. EQUIVALENCE OF STRUTS WITH AND WITHOUT FLEXIBLE RESTRAINTS IN TERMS OF ELASTIC CRITICAL BUCKLING LOAD

Without any lateral restraints, the elastic critical buckling load, Ncrit, of this strut is 173 kN. The presence of the springs increased this to 274 kN and the buckling mode remained in a single half-wavelength between end supports. From the Euler strut buckling equation, the effective strut length, l_e, to give this same value of N_{crit} with no lateral restraints is 7.94 m. A pin ended model of length 7.94 m with no springs was therefore also set up.

The two models were analysed with geometric non-linearity to obtain the axial load at which first yield occurred. An initial imperfection having the shape of the first mode of buckling was applied to the models. The deflections were scaled so that the maximum imperfection offset was equal to l_w/250 = 40 mm for model type (a) (Figure 5) with lateral restraints. L/250 is the imperfection recommended in BS EN 1993-1-1 for second order analysis of this particular strut geometry. Model type (b) was analysed twice, first with a maximum imperfection of l_e/250 = 31.8 mm and then with the same imperfection of 40 mm as used in model (a). The first case represents the Eurocode approach of using buckling curves based on equivalent geometric imperfections appropriate to the effective length and the second case represents the BS 5400 approach using an imperfection factor appropriate to the half wavelength of buckling.

TABLE 1. LOAD AT FIRST YIELD FOR MODELS (A) AND (B)

Model	Imperfection (mm)	N first yield (kN)
(a)	40.0	245
(b)	31.7	239
(b)	40.0	231

Table 1 shows the load at which the outermost fibre of the beam first yielded for each case. Model (a) represents the true resistance of the strut with intermediate restraints. It can be seen that the equivalent shorter strut without restraints represented by model (b) had a lower resistance even when the smaller imperfection based on le was used. This shows that the codified approach in BS EN 1993-1-1 is safe without the need to consider the ratio l_e/l_w.

It is easy to illustrate why model (a) produces the greatest resistance. The first order moment acting on model (b) is (N x a_0) where a_0 is the initial imperfection and N is the axial force. Where lateral restraints are present, the first order moment is lower than this because of the transverse resistance offered. A first order linear elastic analysis of model (a) gives M = (0.65 x N x a_0). In both models, the second order moment considering P-Δ effects can be obtained approximately by increasing the first order moment by a factor of

$$\frac{1}{1 - N / N_{crit}}$$

If a_0 is the same for both cases the smaller first order moment in the case with lateral restraints gives rise to a smaller second order moment and hence a higher ultimate buckling load. Using an imperfection based on the effective length in model (b) still gives a conservative buckling resistance as the first order moment (0.65 x N x a_0) for the restrained strut is still less than that for the effective length strut of (N x a_0 x L_e/L_w) = (N x a_0 x 0.794). This implies that actually the curves in BS EN 1993-1-1 become more conservative for cases of beams with intermediate restraint, rather than less conservative as implied by BS 5400:Part 3.

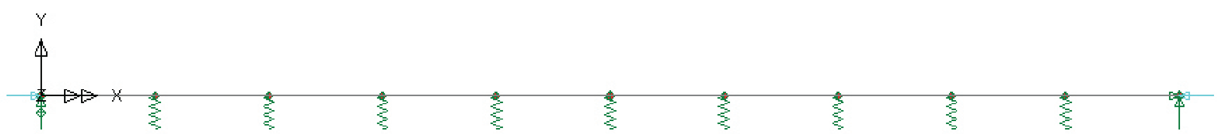


FIGURE 6. MODEL OF STRUT WITH LATERAL RESTRAINTS

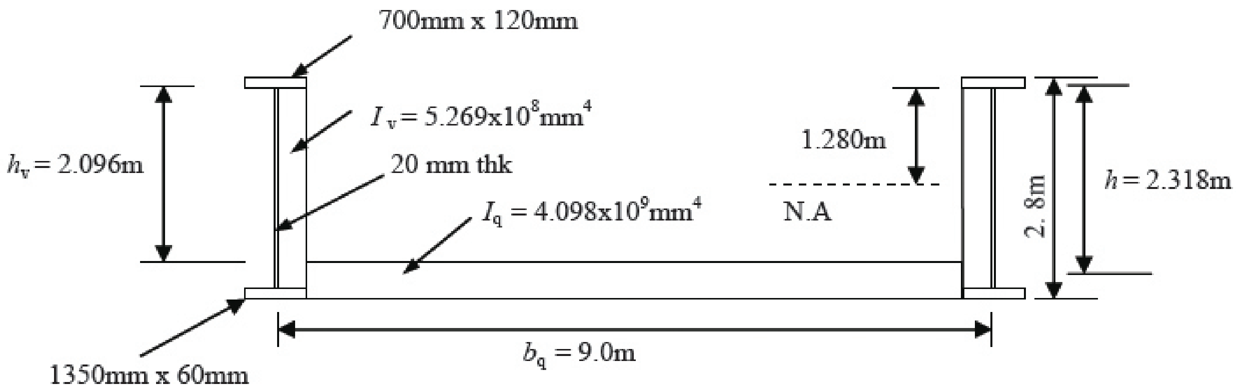


FIGURE 7. SKETCH OF MODEL REPRODUCED FROM DESIGNERS' GUIDE TO EN 1993-2

The above result can also be demonstrated more generally by solution of the governing differential equation for a curved beam on an elastic foundation with axial force, but the reader is spared the mathematics here.

3.2 Half-through bridge with U-frame restraint

A specific case of a half-through bridge in S355 steel was investigated. The bridge is simply supported with a 36 m span and cross girders at 3 m centres. The transverse web stiffeners coincide with the cross girders. The dimensions of the case considered are given in Figure 7 and is based on worked example 6.3-6 in the Designers' Guide to EN 1993-2[4].

The slender main girders are class 4 to EN 1993-1-1 (and non-compact to BS 5400:Part 3). The deck slab is composite with the cross girders but is not fixed to the main girders.

The bridge was modelled using shell elements in the finite element package LUSAS. The layout of the FE model is shown in Figure 8. For simplicity, the deck slab was not included explicitly in the model, other than in the rigidity of the cross members. To prevent relative longitudinal movement between main girders, plan bracing was added to the model at the ends and transverse supports were provided at each cross girder to prevent lateral buckling into the deck slab. Pinned vertical point supports were provided at the end of each girder and longitudinal movement was permitted at one end.

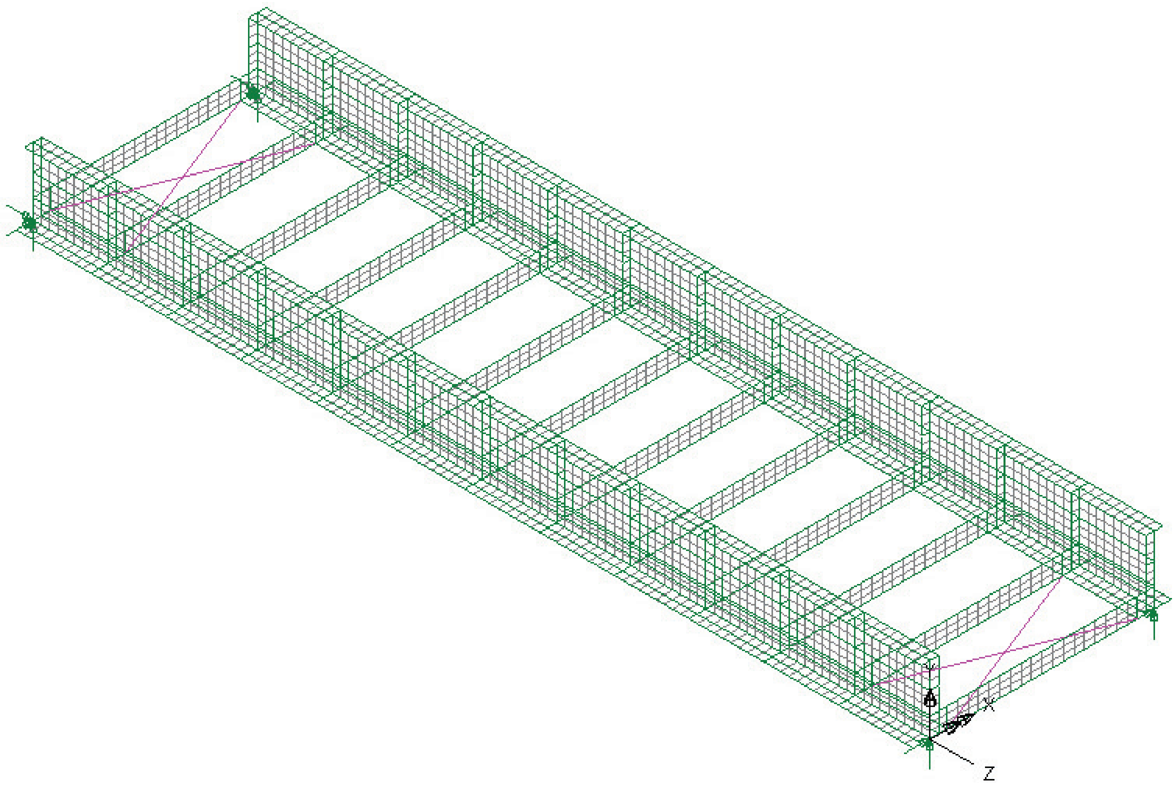
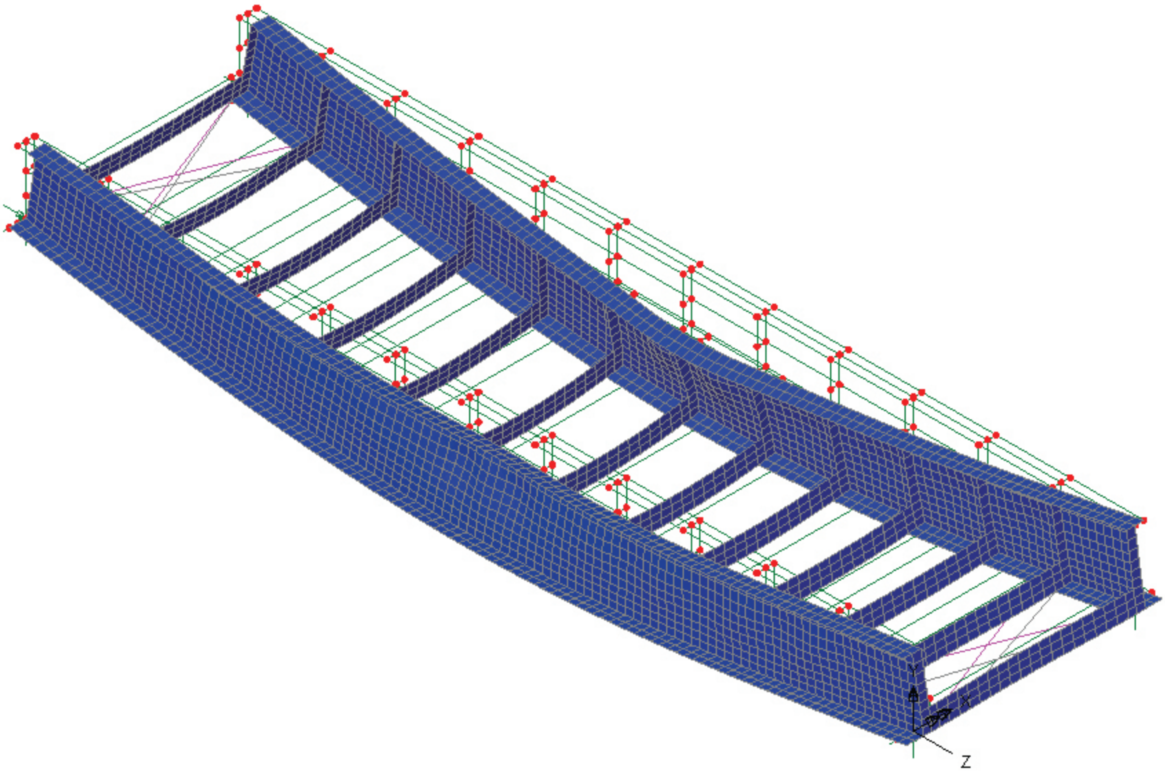


FIGURE 8. FINITE ELEMENT MODEL OF HALF-THROUGH BRIDGE



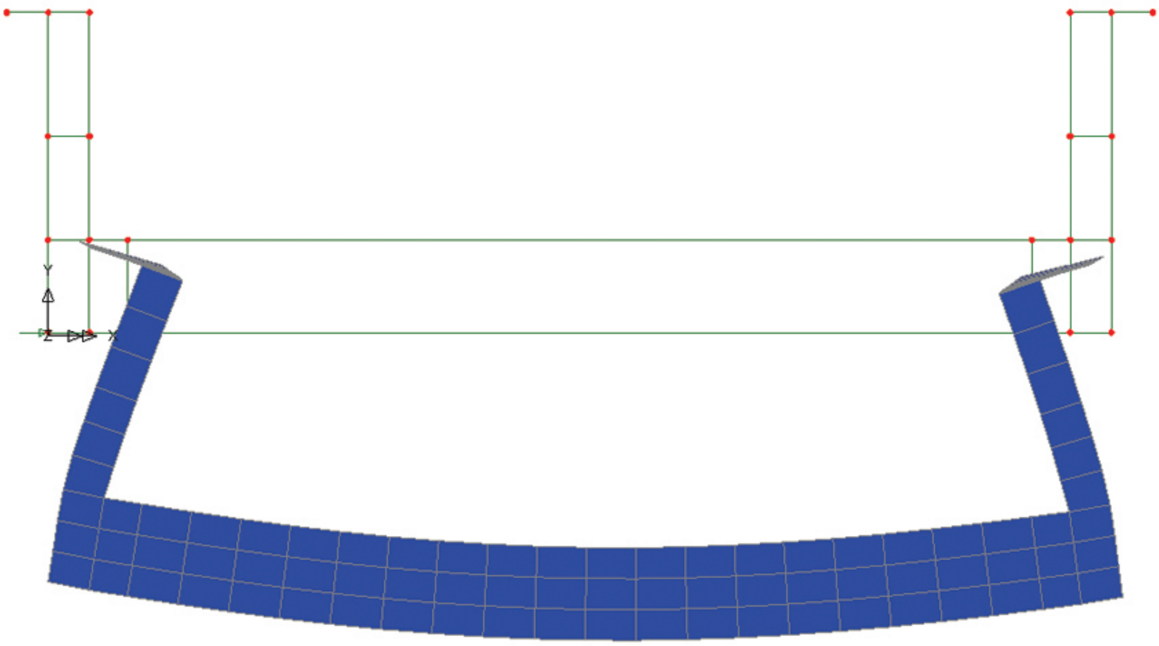


FIGURE 9. DEFLECTED SHAPE AT FAILURE

In order to prevent local failure of the model at the point supports, the stiffener and web plate thicknesses at the end of the model were increased locally. Loading was applied uniformly distributed along the top of the cross girders.

The model was first analysed linear elastically with vertical and lateral load cases to check that deflections and flexural stresses were as expected. The true resistance of the bridge to uniform vertical loading was then determined from a materially and geometrically non-linear analysis. The non-linear material properties used were based on a material model given in EN1993-1-5 Appendix C. In this model yield occurs at a stress of 335 MPa and a strain of 0.001595. To model limited strain hardening, the gradient of the stress-strain curve was then reduced from 210 GPa to 2.10 GPa up to an ultimate strain of 0.05.

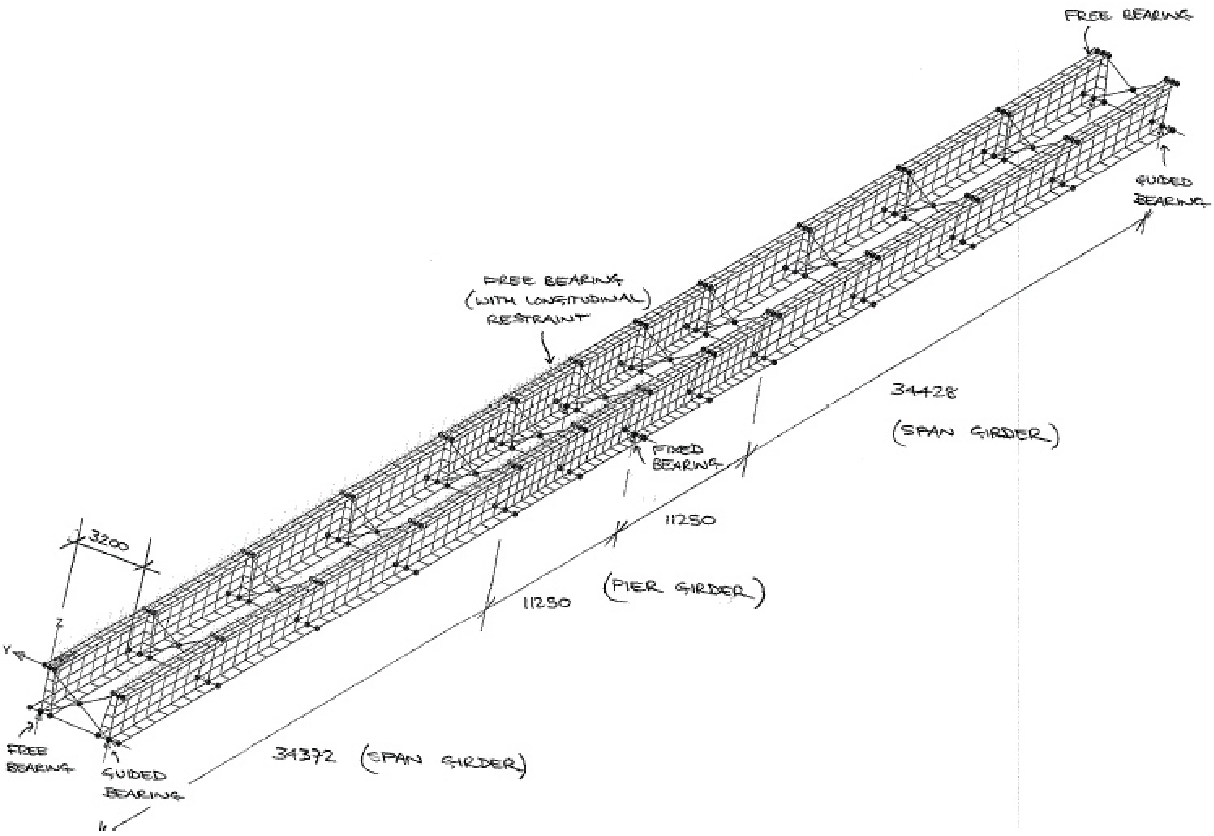


FIGURE 10. IDEALISATION OF TWO SPAN BRACED PLATE GIRDERS

Non-linear analysis

An initial deflected shape similar to the elastic buckling mode expected was generated by applying a point load to the top flanges of the main girders at midspan in a linear analysis. The deflected shape from this analysis was factored so that the inwards deflection at the start of the non-linear analysis had magnitude L/150 where L was taken as the distance between points of contraflexure in the flange. The value L/150 is taken from EN1993-1-1 Table 5.1 and accounts for allowable construction imperfections and residual stresses in the girders.

After the initial analysis to failure, shown in Figure 9, a second non-linear analysis was performed using a modified shape of initial imperfection based on a scaled version of the deflections at failure from the first analysis. The deformed shape was scaled so that the horizontal deflection at the top flange was again L/150. This gave a moment of resistance of 72340 kNm.

Hand calculations for buckling resistance in accordance with BS EN 1993-2[6] gave a buckling moment of resistance as 54360 kNm (without any partial factors). The non-linear FE analysis therefore gave 33% more resistance than the code calculations. For comparison, the elastic moment resistance of the girder was 77988 kNm and the plastic moment resistance of the girder was 86373 kNm.

In order to produce a more pronounced buckling failure, the cross bracing and stiffener spacing was increased to 6 m. The same analysis procedure as for the first model was repeated. The model failed at an ultimate applied moment of 62076 kNm. The hand calculations were repeated using the reduced stiffness of the U-frame. The new buckling moment was found to be 52252 kNm. This is 19% lower than that found in the non-linear analysis.

The analyses both show that the non-linear FE models demonstrate considerably more strength than is predicted by the method in BS EN 1993-2 using the buckling curves in BS EN 1993-1-1. The model with girders at 3 m centres gives 33% extra resistance and the model with girders at 6 m centres gives 19% extra resistance. Once again, the buckling curves of BS EN 1993-1-1 were found to be conservative and thus the BD 13/06 approach of using multiple curves to allow for the ratio l_e/l_w is unnecessary

There are several reasons why the non-linear FE model gave higher predicted strength than the calculations to EN 1993. These include:

1) The FE model shows partial plastification of the tension zone occurs, which gives extra resistance that is not accounted for in the hand calculations.
2) The strain hardening included in the material properties allows the stresses in the model to increase beyond yield (by roughly 7%).

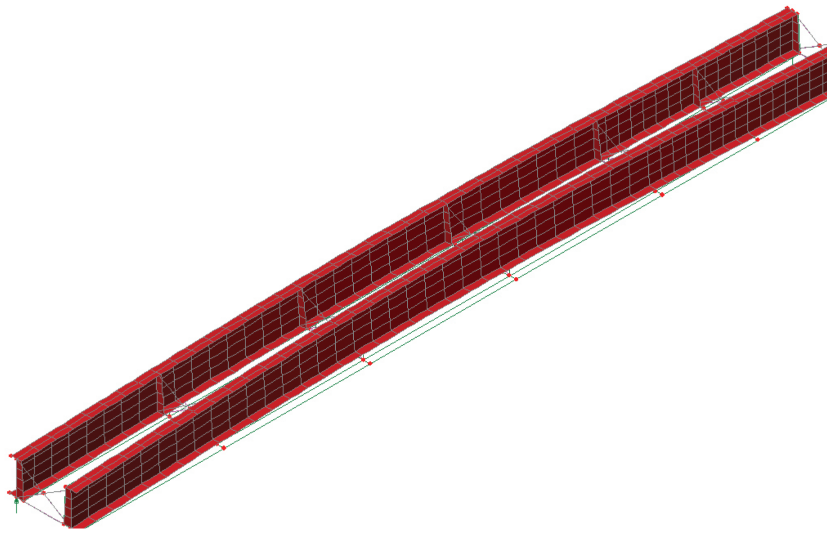


FIGURE 11. LOWEST GLOBAL MODE OF BUCKLING SINGLE SPAN BEAMS

3.3 Paired beams during construction

The composite bridge cases considered were a two span and a single span bridge with two steel plate girders braced together by cross bracing. The dimensions are representative of typical UK construction, being based on a recently constructed bridge. Figure 10 shows the geometry of the two span bridge and the FE model setup and Table 2 gives the dimensions of the girder. A uniformly distributed vertical load was applied to both girders in one span only, representing concreting of a single span.

TABLE 2. GIRDER MAKE-UP

SPAN GIRDER	Width	Depth	f_y (MPa)
Top Flange	600	40	345
Web	16	1942	355
Bottom Flange	810	33	345

Span Girder

PIER GIRDER	Width	Depth	f_y (MPa)
Top Flange	600	59	345
Web	16	1942	355
Bottom Flange	810	59	345

Pier Girder

The bridge layouts were checked for lateral torsional buckling during construction using four different approaches:

a) The standard method set out in BD 13/06 9.6.4.1.2 was followed to obtain the slenderness λ_{LT} and then the resistance moment from Figure 11

b) The alternative method permitted in clause 9.7.5 of BD13/06 was used to obtain λ_{LT} from a value of M_{cr} determined from an elastic critical buckling analysis using the FE model. The effective length was back-calculated from λ_{LT} using clause 9.7.2 and the resistance moment obtained from Figure 11 of BD 13/06

c) EN 1993-1-1 clause 6.3.2 was used to calculate the slenderness

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_y}{M_{cr}}}$$

(where M_{cr} was obtained from an elastic critical buckling analysis and M_y was the first yield moment)

d) Non-linear analysis

With a continuous bridge, redistribution of moment away from the span to the support is possible with a non-linear analysis when the mid-span region loses stiffness through buckling. This would make the conclusions from the comparison of ultimate load obtained for continuous spans with the code approaches in a) to c) inapplicable to simply supported spans where such redistribution could not occur. The above approaches were therefore repeated for a single span model, with the same dimensions as half the two span bridge, but with the span girder properties used throughout. For the single span bridge, load was applied to both girders over the whole span.

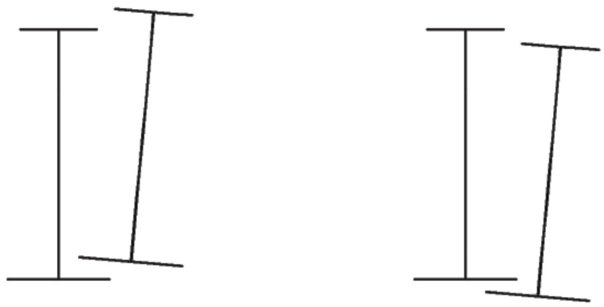


FIGURE 12. LOWEST GLOBAL MODE OF BUCKLING FOR SINGLE SPAN BEAMS – ROTATION OF CROSS-SECTION

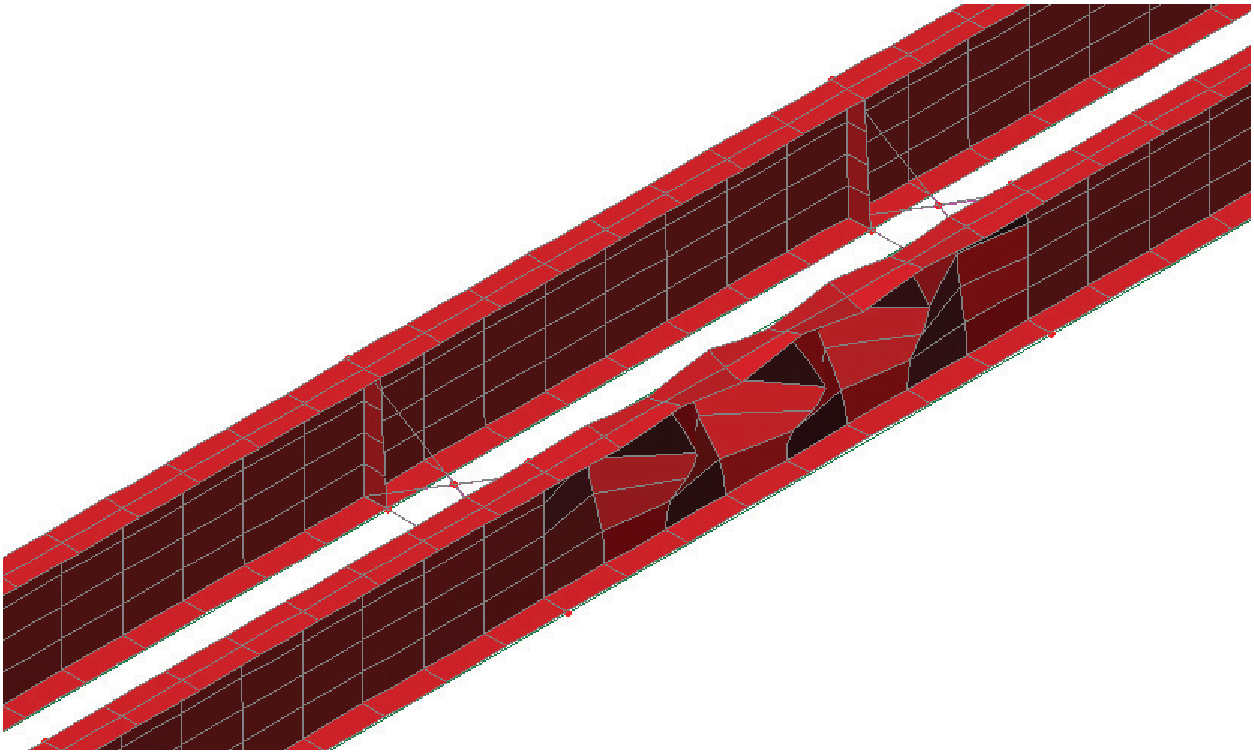


FIGURE 14. TYPICAL LOCAL
BUCKLING FOR SINGLE SPAN BEAMS

FIGURE 13. SECOND LOWEST GLOBAL MODE
OF BUCKLING FOR SINGLE SPAN BEAMS

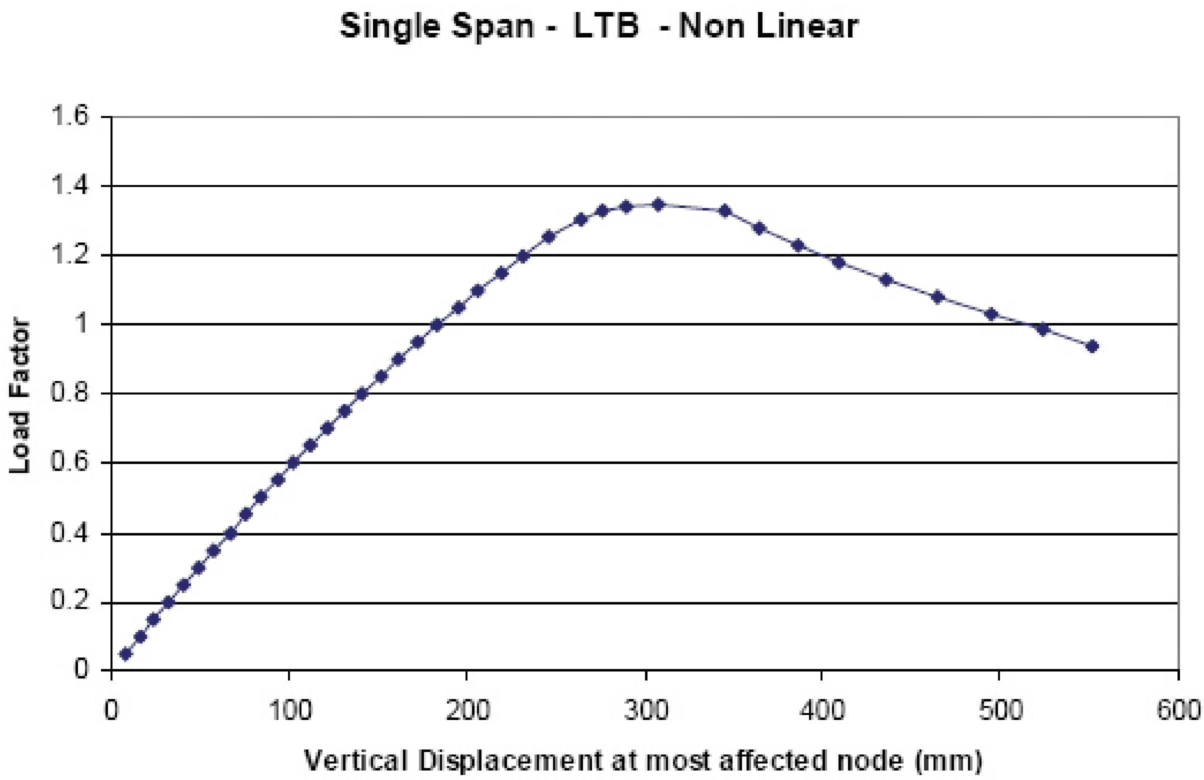


FIGURE 15. LOAD-DEFLECTION CURVE FOR NON-LINEAR ANALYSIS OF SINGLE SPAN MODEL

Elastic critical buckling analysis

The lowest global mode of buckling for the one span model, corresponding to the attainment of M_{cr} , is shown in Figures 11 and 12. The girder pair is seen to rotate together over the whole span as illustrated in Figure 2. The second lowest mode is shown in Figure 13 and corresponds to lateral buckling of the compression flange between braces. At lower load factors than either of these modes, a number of local buckling modes such as that shown in Figure 14 were found. These typically correspond to buckling of the top of the web plate in compression or potentially to torsional buckling of the top flange and may be ignored for the purposes of determining Mcr; these buckling effects are considered in the section properties in codified approaches to BS 5400:Part 3 and BS EN 1993. The values of M_{cr} determined were used to derive a total resistance moment for cases b) and c).

TABLE 3. BENDING RESISTANCES OBTAINED FOR THE DIFFERENT METHODS

Calculation Method	Design Resistance Moment at mid-span (Including γ_m and γ_{f3} on the resistance side) (kNm)	
	Two Span	Single Span
a) BD 13/06 9.6.4.1.2	6045 (l_e = 17.0m)	5260 (l_e = 18.9m)
b) BD 13/06 9.7.5 (with FE)	7330 (l_e = 13.5m)	6085 (l_e = 16.0m)
c) EN 1993-1-1 6.3.2 (with FE)	9231	7470
d) Non Linear FE (maximum M)	12000	9591
d) Non Linear FE (first yield)	9654	8170

Non-linear analysis

The same FE models for single span and two span cases were analysed considering non-linear material properties and geometry and including an initial deformation corresponding to the first global buckling mode. This was used to determine the collapse load. The magnitude of the initial deflection was taken as L/150 as required in EN 1993-1-1. The maximum moment reached and the moment at which first yield occurred were noted. Failure occurred by rotation of the braced pair over a span in the same shape as the elastic buckling mode of Figure 11. Figure 15 shows the load-deflection curve up to failure for the single span model. It indicates that the failure is reasonably ductile.

The design resistance moments for the two span and single span models are given in Table 3. The resistances are all factored up by the partial material factors γ_m and γ_{f3} in BD 13/06 so that they are all directly comparable and, for the two span beam non-linear cases, are based on the original elastic bending distribution along the paired beams without allowing for any moment redistribution away from the span. (This was achieved by using the load factor at collapse to scale up the original elastic moments.) The effective length used in the code calculations is also given. The ratio l_e/l_w lies between 0.3 and 0.5 and Figure 3 shows that this will give a significant reduction in strength in the BD 13/06 calculations.

The non-linear analysis gave higher resistance moments, thus showing that methods a) to c) were all conservative with the methods of a) and b) based on BD 13/06 being the most conservative. This again indicates that the BD 13/06 approach of using multiple curves to allow for the ratio l_e/l_w is unnecessary. Again, there were several reasons why the non-linear FE gave higher predicted strength than the other code methods:

1) Partial plastification of the tension zone is possible.
2) Strain hardening can occur.
3) For the two span model, redistribution of elastic moment is possible away from the span.

4. Conclusions

All the analyses demonstrate that the resistance curves and slenderness calculation used in Eurocode 3 are conservative. The analyses of the two simple strut cases show that applying the initial imperfection relevant to a length equal to the half wavelength of buckling, where this is greater than the effective length, is overly conservative. The curves in BD 13 and BS 5400 Part 3 therefore need to be revised to remove the curves below that for le/lw = 1.0. The curves for le/lw = 1.0 can always be used safely.

Non-linear analysis can be used to extract greater resistance from beams for a number of reasons which include benefit from:

- partial plastification of the tension zone in non-compact sections
- strain hardening
- moment redistribution in statically indeterminate structures

The case for including the multiple buckling curves in BS 5400: Part 3 and BD 13/06 was challenged by the authors previously and this eventually resulted in the Highways Agency and the British Standards Institutions (BSI) B/525/10 committee accepting that the lower l_e/l_w curves were not appropriate. The BSI does not intend to revise BS 5400: Part 3 but the B/525/10 committee agreed that the revision, removing the lower curves, can be published by the Steel Constructions Institute. The revision was recently published in the New Steel Construction as advisory desk not AD 326.[7]

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This paper will be published in the Proceeding of the ICE Bridge Engineering, March 2009